1. Suppose that \( a \) and \( b \) are integers such that \( a + 2b \) and \( b + 2a \) are squares. Prove that each of \( a \) and \( b \) is a multiple of 3.

2. In the figure, \( P \) is a point on the circumcircle of \( \triangle ABC \). Lines \( \overline{AX} \) and \( \overline{CY} \) are drawn so that \( \angle PAC = \angle BAX \) and \( \angle PCA = \angle BCY \). Prove that \( \overline{AX} \) and \( \overline{CY} \) are parallel.

3. (NEW YEAR’S PROBLEM) Let us write \( P(n) \) to denote the smallest prime number that does NOT divide \( n \) and use \( Q(n) \) to denote the product of all prime numbers less than \( n \), with \( Q(2) \) defined to be 1. Construct a sequence of numbers \( X_n \) as follows. Put \( X_0 = 1 \) and for each integer \( n > 0 \), define \( X_n = X_{n-1}P(X_{n-1})/Q(P(X_{n-1})) \). Thus the first several numbers in this sequence are 1, 2, 3, 6, 5, 10, 15, 30, 7, . . . . Compute \( X_{1998} \).

4. Prove that the average of the squares of three real numbers can never be less than the square of the average of these numbers.

5. Find all polynomial functions \( F(x) \) such that \( F(0) = 2 \) and \( F(x^2 + 1) = F(x)^2 + 1 \) for all \( x \).