1. Given two real numbers $a$ and $b$, suppose that the average of their fourth powers is equal to the fourth power of their average. Show that the two numbers must be equal.

2. Let $I$ be the center of the inscribed circle of $\triangle ABC$. Show that the center of the circumscribed circle of $\triangle BIC$ lies on the circumscribed circle of $\triangle ABC$.

3. Recall that a rational number is one that can be written in the form $m/n$, where $m$ and $n$ are integers. Suppose that $a$, $b$ and $c$ are positive rational numbers and that $\sqrt{a} + \sqrt{b} + \sqrt{c}$ is also rational. Show that $\sqrt{a}$, $\sqrt{b}$ and $\sqrt{c}$ are each rational.

4. Suppose that all the integers $n > 1000$ are divided into two sets $A$ and $B$. Show that at least one of these sets contains two different numbers $x$ and $y$ such that $x + y$ is also in that set.

5. Recall that the Fibonacci numbers are $1, 1, 2, 3, 5, 8, \ldots$, where after the first two, each is the sum of the preceding two numbers. Write $F_n$ to denote the $n$th Fibonacci number and let $r$ denote the number $(1 + \sqrt{5})/2$. Prove that the ratio $F_{100}/F_{99}$ is so close to $r$ that the difference satisfies $|F_{100}/F_{99} - r| < 10^{-20}$.