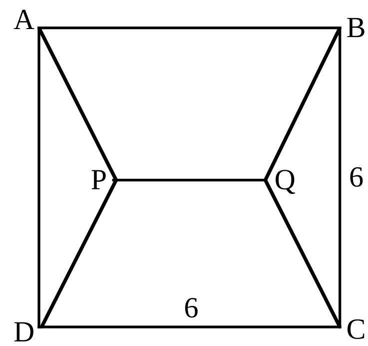
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (1997-98) **NOVEMBER 1997**

1. Suppose that the quintic equation $x^5 - ax + b = 0$ has two different positive solutions for x. Show that there is also some negative value of x that makes the equation true.

2. We wish to place six points within a 6×6 square so that the minimum distance between any two of the points is as large as possible. For example, we could place the points at A, B, C, D, P and Q in the diagram with $PA = PD = PQ = QB = QC$. Compute the minimum distance in this case and find a configuration in which the minimum distance between points is even larger.



3. Find all square numbers of the form $n^2 + n + 43$ with n a nonnegative integer.

4. Consider the following two-person game. We start with three piles of 10 coins each, with one pile designated as the "hot" pile. On his or her turn, a player can take any positive number of coins from any non-hot pile and put those coins on any other pile. The destination pile then becomes the new hot pile. Note that no move is possible if all 30 coins are in the hot pile. The game is over when some player cannot make a move, and in that case, that player loses. Prove that the second player can always force a win.

5. Observe that the number $n = 536$ has no repeated digits and that its double, $2n = 1072$ also has no repeated digits and has no digit in common with n. Find the largest integer n such that n and $2n$ have no repeated digits and have no digits in common.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH	DEADLINE
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	University of Wisconsin, Madison, WI 53706	1997

(Please Detach)

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PROBLEM	SCORE
1	
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PROBLEM SET II