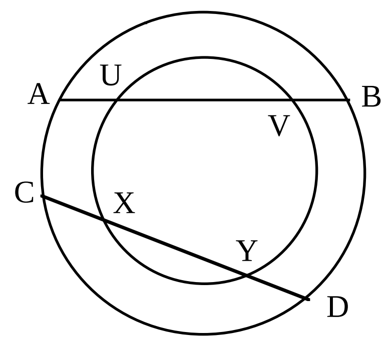
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (1997-98) **OCTOBER 1997**

1. Everyone has just one "magic birthday", when his age is exactly equal to the sum of the digits of the year of his birth. For example, the magic birthday of someone born in 1899 was in 1926. Notice that someone born in 1908 also had a magic birthday in 1926. What is the next year after 1926 in which two people born in different years can both have magic birthdays.

2. Suppose chords $\overline{AB}$ and $\overline{CD}$ of a circle meet a smaller concentric circle at points U, V, X and Y, as shown. If $AU = 2$, $UV = 10$ and $CX = 3$, find XY and prove that your answer is correct.



3. Recall that for each positive integer n, we write $n!$ to denote the product $1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$. (This is called "$n$ factorial".) Prove that $n!$ can never be a multiple of 2^n for any positive integer n.

4. Let $a < b < c < d < e$ be real numbers and let S be the set of all possible sums obtained by adding two distinct numbers from these five. If S has only seven members, show that a, b, c, d and e form an arithmetic progression.

5. Let x and y be positive real numbers satisfying $x^3 + y^3 = 2xy$. Show that $x < 2^{2/3}$ and $y < 2^{2/3}$. (This is, in fact, not the best possible bound.)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE November 3 1997

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(Please Detach)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
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4	
5	

PROBLEM SET I