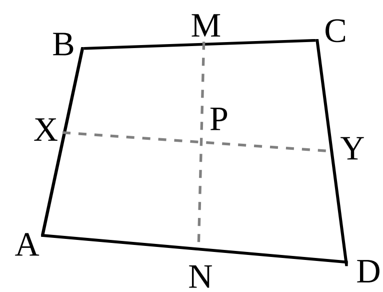
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (1996-97) **JANUARY 1997**

1. Your calculator will tell you that $\sqrt[3]{6\sqrt{3}+10} - \sqrt[3]{6\sqrt{3}-10}$ is approximately equal to 2. Is this quantity exactly equal to 2? Prove that your answer is correct.

2. Let $ABCD$ be a quadrilateral and let X, M, Y and N be respectively the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$. Show that the point P where $\overline{XY}$ and $\overline{MN}$ meet is the midpoint of each of $\overline{XY}$ and $\overline{MN}$.



3. (NEW YEAR'S PROBLEM) Let m and e be positive integers and suppose that $N = 1997m/(m + 1997^e)$ is an integer. Find all possible values for N.

4. I have a magic money machine into which I can put any number of one dollar coins. If I insert n dollars, the machine returns $2n$ dollars. Each time I use the machine, however, I must insert more money than I did on the previous use. If I start with exactly \$1 and use the machine once, I will have \$2. One my next use of the machine, I am forced to insert \$2 yielding \$4, and on my third use of the machine, I can insert either \$3 or \$4 yielding a total of \$7 or \$8. Consequently, there is no way that I can ever obtain exactly \$3 or \$5 or \$6 by using the machine repeatedly, starting with \$1. Find the largest integer L such that it is impossible to obtain exactly L dollars with the magic money machine, starting with \$1.

5. Let S be a subset of the set $\{1, 2, 3, \ldots, 1000\}$ with the property that no sum of two distinct members of S is contained in S. Find the maximum possible number of members in the set S.

You are invited to submit a solution even if you get just one problem

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RETURN TO:	MATHEMATICS TALENT SEARCH	DEADLINE
	Dept. of Mathematics, 480 Lincoln Drive	February 14
	University of Wisconsin, Madison, WI 53706	1997

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(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

HOME ADDRESS TOWN ZIP CODE

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET IV