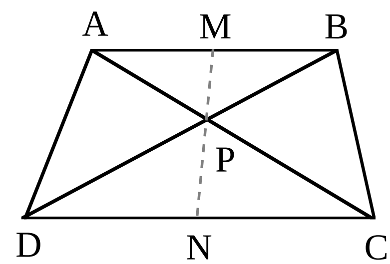
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (1996-97) **DECEMBER 1996**

1. Let $\square$ be a binary operation defined on the set of real numbers. (This means that if x and y are any two real numbers, then $x \square y$ is a real number determined by x and y.) Suppose that $(a \square b) + c = (b + c) \square (a + c)$ and $0 \square (a + b) = (0 \square a) + (0 \square b)$ for all real numbers a, b and c. Compute $23 \square 77$ and prove that your answer is correct.

2. The diagonals of quadrilateral $ABCD$ meet at point P, as shown. When the midpoints M of $\overline{AB}$ and N of $\overline{DC}$ are joined, it is found that the line $\overline{MN}$ goes through the point P. Prove that $\overline{AB}$ and $\overline{DC}$ are parallel.



3. Let a, b and c be positive real numbers and suppose that $a^m + b^m = c^m$ and $a^n + b^n = c^n$, where m and n are also positive real numbers. Show that $m = n$.

4. I have a magic money machine that does the following. When I put in a nickel, the machine gives back two dimes. When I put in a dime, out pop a quarter and three nickels. Inserting a quarter yields a one dollar coin and finally, if I put in a dollar coin, the machine returns three dimes and two nickels. Starting with just one dollar coin, and using the money machine repeatedly, is it possible to end up with coins worth exactly $50?

5. One solution for the equation $a^2 + b^2 + c^2 + 2 = abc$ is $a = 3$, $b = 3$ and $c = 4$. Do there exist integer solutions of this equation with a, b and c all larger than 10? Either find such a solution or prove that none exists.

You are invited to submit a solution even if you get just one problem

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RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE January 15 1997

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(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

HOME ADDRESS TOWN ZIP CODE

PROBLEM	**SCORE**
1	
2	
3	
4	
5	