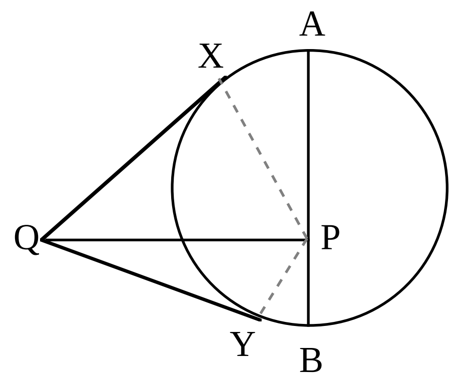
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (1996-97) **NOVEMBER 1996**

1. If a, b and c are nonzero real numbers, show that $a+b+c$ and $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ cannot both be zero.

2. In the figure, $\overline{AB}$ is a diameter of the circle and $\overline{QP}$ is perpendicular to $\overline{AB}$. Also, $\overline{QX}$ and $\overline{QY}$ are the two tangents to the circle from Q. Show that $\overline{QP}$ bisects $\angle XPY$.



3. If x and y are integers (not necessarily positive) satisfying

$$x + x^2 + x^8 = y + y^2 + y^8$$

prove that $x = y$.

4. A positive integer $x = d_n d_{n-1} \ldots d_1 d_0$ is said to be *special* if the digits d_i in its decimal expansion satisfy

$$d_n \leq d_{n-1} \leq \cdots \leq d_1 \leq d_0 = 5.$$

For example, both 15 and 225 are special. Show that there are infinitely many integers x such that both x and x^2 are special.

5. For each integer $k \geq 3$, prove that it is possible to write 1 as the sum of the reciprocals of k distinct positive integers.

You are invited to submit a solution even if you get just one problem

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RETURN TO:	MATHEMATICS TALENT SEARCH	DEADLINE
	Dept. of Mathematics, 480 Lincoln Drive	December 2
	University of Wisconsin, Madison, WI 53706	1996

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(PLEASE DETACH)

LAST NAME FIRST GRADE

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PROBLEM	SCORE
1	
2	
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