1. If \(a, \ b, \text{ and } c\) are nonzero real numbers, show that \(a + b + c\) and \(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\) cannot both be zero.

2. In the figure, \(AB\) is a diameter of the circle and \(QP\) is perpendicular to \(AB\). Also, \(QX\) and \(QY\) are the two tangents to the circle from \(Q\). Show that \(QP\) bisects \(\angle XPY\).

3. If \(x\) and \(y\) are integers (not necessarily positive) satisfying
\[x + x^2 + x^8 = y + y^2 + y^8\]
prove that \(x = y\).

4. A positive integer \(x = d_n d_{n-1} \ldots d_1 d_0\) is said to be special if the digits \(d_i\) in its decimal expansion satisfy
\[d_n \leq d_{n-1} \leq \cdots \leq d_1 \leq d_0 = 5.\]
For example, both 15 and 225 are special. Show that there are infinitely many integers \(x\) such that both \(x\) and \(x^2\) are special.

5. For each integer \(k \geq 3\), prove that it is possible to write 1 as the sum of the reciprocals of \(k\) distinct positive integers.

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You are invited to submit a solution even if you get just one problem