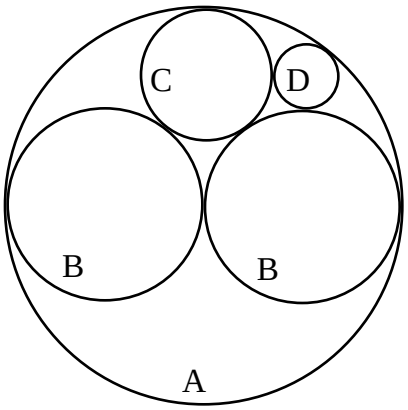
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (1995-96) **FEBRUARY 1996**

1. Let n be a positive integer and let x, y and z be real numbers with x and y both ≥ 1. Suppose $x^n + y^n = z^n + 1$ and $x + y = z + 1$. Show that at least one of x, y or n is equal to 1.

2. In the figure we see five circles. The largest of these (labeled A) has radius r. Each of the two circles labeled B has radius $r/2$, and they are tangent to each other and to A. Circle C is tangent to A and to both circles B and, as we saw in Problem Set III, C has radius $r/3$. Finally, circle D is tangent to A, C and to one of the circles B. Compute its radius.



3. I have a multidigit number n whose units digit is a 1. My friend copied the number, but accidently transposed two adjacent digits to obtain the number m different from n. Find the largest possible integer which can be a divisor of both n and m.

4. Find all positive integers n such that each of the three numbers $6n^2 + 5$, $2n^2 + 3$ and $n^2 + 1$ is prime.

5. Recall from a previous problem set that a cut-and-flip operation on a deck of 52 cards is the process of removing the top k cards from the deck, inverting this stack of k cards, and then replacing the inverted stack on the top of the deck. If all the cards in the deck are face down, show that it is possible to find a sequence of cut-and-flip operations that interchanges the 19th and 20th cards (counting from the top), leaves all the other cards in their original positions, and leaves all the cards face down.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE March 12 1996

(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

HOME ADDRESS TOWN ZIP CODE

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET V