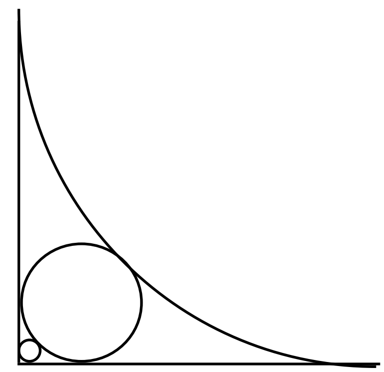
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (1995-96) **JANUARY 1996**

1. Consider the following sequence of numbers: 4, 12, 32, 80, 192, ..., where the formula for the nth number is $(n+1)\,2^n$. Find a formula for the average of the first n numbers in the sequence.

2. In the figure, a circle of radius 1 is tangent to two perpendicular lines. A second smaller circle is drawn tangent to the first circle and to the two lines, and a third circle is tangent to the second and the two lines. Imagine continuing this process until a total of ten circles have been drawn. Find the radius of the tenth circle.



3. Given a positive integer $B \neq 6$, show that it is possible to find a prime number p so that $9p^2 + Bp + 1$ is not a perfect square.

4. Recall that if n is a positive integer, then $n!$ is the product of all the integers from 1 to n, inclusive. Show that $(2^{10})! > 2^{2^{13}}$.

5. For each integer $n \geq 0$, we have a rule which gives a new integer, denoted by n^*. Suppose that

$$\frac{(n+1)^* + (n-1)^*}{2} = n^* + 1$$

for all $n \geq 1$. If $0^* = 0$ and $100^* = 20{,}000$, find 200^*.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE February 6 1996

(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

HOME ADDRESS TOWN ZIP CODE

PROBLEM	SCORE
1	
2	
3	
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5	

PROBLEM SET IV