1. Consider the following sequence of numbers: 4, 12, 32, 80, 192, ..., where the formula for the $n$th number is $(n + 1)2^n$. Find a formula for the average of the first $n$ numbers in the sequence.

2. In the figure, a circle of radius 1 is tangent to two perpendicular lines. A second smaller circle is drawn tangent to the first circle and to the two lines, and a third circle is tangent to the second and the two lines. Imagine continuing this process until a total of ten circles have been drawn. Find the radius of the tenth circle.

3. Given a positive integer $B \neq 6$, show that it is possible to find a prime number $p$ so that $9p^2 + Bp + 1$ is not a perfect square.

4. Recall that if $n$ is a positive integer, then $n!$ is the product of all the integers from 1 to $n$, inclusive. Show that $(2^{10})! > 2^{2^{13}}$.

5. For each integer $n \geq 0$, we have a rule which gives a new integer, denoted by $n^*$. Suppose that
\[
\frac{(n + 1)^* + (n - 1)^*}{2} = n^* + 1
\]
for all $n \geq 1$. If $0^* = 0$ and $100^* = 20,000$, find $200^*$.