

WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (1995-96)

DECEMBER 1995

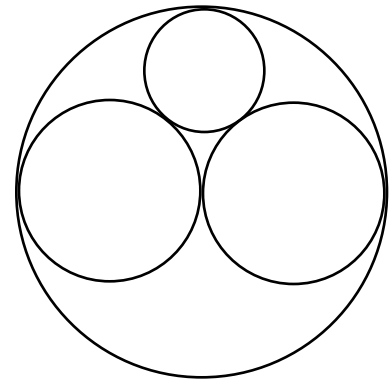
1. A deck of 52 cards has some cards face up and some face down. We are allowed to rearrange this deck using a sequence of cut-and-flip operations. Specifically, a cut-and-flip operation is achieved by removing a stack of cards from the top of the deck, turning this stack over, and then returning the inverted stack to the top of the deck. Note that turning over the entire deck or just turning over the top card are examples of cut and flip operations. Prove that it is possible to get all the cards face down using no more than 52 such operations.

2. Shown here are four circles, each tangent to the other three. The largest of these has radius 2 and each of the two medium-sized circles has radius 1. Find the radius of the smallest circle and justify your answer.

3. Find all positive integers which have precisely 36 positive divisors and which are divisible by precisely nine of the numbers from 1 to 10.

4. (New Year's Problem) Find all positive integers n for which $\sqrt{n + \sqrt{1996}}$ exceeds $\sqrt{n - 1}$ by an integer.

5. Consider the equation $(x^2 + 10x + a)^2 = b$. (i) If $a = 21$, find a number b so that there are exactly three real values of x which satisfy the equation. (ii) If $a \geq 25$, show that there are no numbers b for which the equation has exactly three solutions.



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
January 2
1996

.....
(PLEASE DETACH)

LAST NAME	FIRST	GRADE
-----------	-------	-------

SCHOOL	TOWN
--------	------

HOME ADDRESS TOWN ZIP CODE

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET III