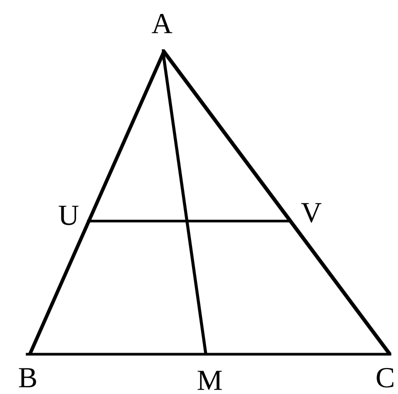
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (1995-96) **NOVEMBER 1995**

1. Sam and Pat keep giving money to each other. They start with a total of \$64,000 between them, and each gift is exactly enough to double the assets of the recipient. They find that eventually Sam has possession of all the money. If each player started with a whole number of dollars, show that each of these numbers must have been a multiple of 125.

2. Let U and V be points on sides $\overline{AB}$ and $\overline{AC}$ of $\triangle ABC$ as shown. Suppose that the median $\overline{AM}$ bisects $\overline{UV}$. Prove that $\overline{UV}$ is parallel to $\overline{BC}$.



3. Find all pairs of positive integers x and y such that $x^2 + y$ exceeds $x + y^2$ by precisely 10.

4. Four towns are located at the vertices of a 4 mile by 6 mile rectangle. By using three sides of the rectangle, a road network of total length 14 miles can be constructed which connects all four towns. Is there a shorter road network which connects the towns? Specifically, is there a suitable network of length less than 13 miles?

5. Let f be a rule which assigns a nonnegative integer called $f(x)$ to each positive integer x. We say that f is a "product-sum rule" if $f(xy) = f(x) + f(y)$ for all positive integers x and y. (a) Find a product-sum rule f such that $f(x)$ is never zero when $x > 1$. (b) If f is a product-sum rule, show that there exist distinct positive integers x and y such that $f(x) = f(y)$.

You are invited to submit a solution even if you get just one problem

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RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
December 1
1995

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(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

HOME ADDRESS TOWN ZIP CODE

PROBLEM	**SCORE**
1	
2	
3	
4	
5	

PROBLEM SET II