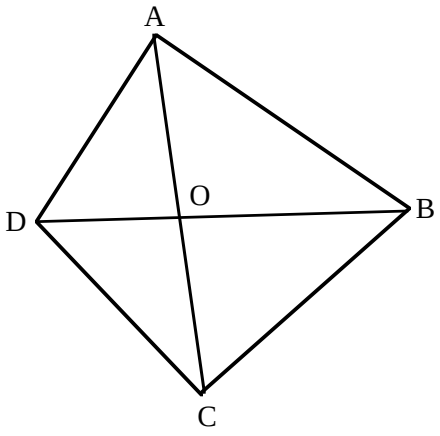
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (1994-95) **JANUARY 1995**

1. Let $\square$ be an operation (like addition or multiplication) which associates to each pair x, y of real numbers the real number $x \square y$. Suppose that, for all real x, y, z, we have (1) $x \square x = x$, (2) $x \square y = y \square x$, (3) $x \square (y \square z) = (x \square y) \square z$, and (4) if $y < z$ and $x \square y \neq x$, then $x \square y < x \square z$. Show that $x \square y = x$ or y for all x, y. Furthermore, find two different operations $\square$ satisfying the above four conditions.

2. In quadrilateral $ABCD$, show that $\angle CAD = \angle CBD$ if and only if $\angle ABC + \angle ADC = 180^\circ$.



3. Solve the equation

$$x^4 + 1 = 2x(x^2 + 1).$$

4. Suppose n is a positive integer. Find the smallest positive integer x such that 2^n divides

$$x^{1995} + 1.$$

5. Find all 3-digit numbers m which are equal to the arithmetic mean of the six numbers one obtains by rearranging the digits of m in all possible ways.

You are invited to submit a solution even if you get just one problem

---

RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
February 6
1995

. . . . . .

(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

HOME ADDRESS TOWN ZIP CODE

PROBLEM	SCORE
1	
2	
3	
4	
5	