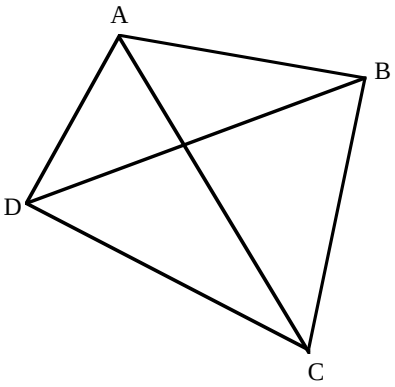
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (1994-95) **OCTOBER 1994**

1. Let $a + 0d,\ a + 1d,\ a + 2d,\ a + 3d,\ \ldots$ be the arithmetic progression determined by the positive integers a and d. Show that this sequence either contains no perfect squares or it contains infinitely many perfect squares.

2. Suppose that the diagonals AC and BD each divide the quadrilateral $ABCD$ into two triangles of equal area. Prove that $ABCD$ is a parallelogram.



3. Prove that
$(x + y + z + w)^2 \geq (8/3)(xy + xz + xw + yz + yw + zw)$

for all real numbers $x,\ y,\ z$, and w.

4. If m is a positive integer, can $m(m + 1)$ be the 7th power of an integer?

5. Let us define a process which replaces each triple of real numbers $t = (a, b, c)$ by a new triple $t' = (a', b', c')$ where $a' = a + b,\ b' = b + c$, and $c' = c + a$. Suppose we start with a triple and apply this process again and again. Show that if we ever return to the original triple, then we will return after just six steps.

You are invited to submit a solution even if you get just one problem

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RETURN TO:	TALENT SEARCH, Dept. of Math. Rm 218 Van Vleck University of Wisconsin, Madison, WI 53706	DEADLINE November 1 1994

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(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

HOME ADDRESS TOWN ZIP CODE

PROBLEM	SCORE
1	
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