WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2014-2015) **February 2015**

1. Let $x_1, x_2, \ldots, x_{12}$ be positive numbers. Show that at least one of the following must be true:

$$\frac{x_1}{x_2}+\frac{x_3}{x_4}+\frac{x_5}{x_6}+\frac{x_7}{x_8}+\frac{x_9}{x_{10}} \geq 5, \quad \frac{x_{11}}{x_{12}}+\frac{x_2}{x_1}+\frac{x_4}{x_3}+\frac{x_6}{x_5} \geq 4, \quad \text{or} \quad \frac{x_8}{x_7}+\frac{x_{10}}{x_9}+\frac{x_{12}}{x_{11}} \geq 3.$$

2. There is an infinite region bounded by two parallel lines containing infinitely many non-overlapping (possibly tangent) circles of radius 1. Suppose that every line perpendicular to the boundary lines intersects or is tangent to at least two of the circles. Find the minimum possible distance between the boundary lines. (Make sure you prove that the minimum you claim is achievable, and that there is no possible smaller width.)

3. Find all right angle triangles where the hypotenuse has length $\sqrt{2} \cdot 2^{2015}$, and the other two sides have integer lengths.

4. Three French farmers, four English farmers, and five Spanish farmers are all attending the International Farmers Conference. They will sit in twelve chairs numbered 1 to 12 equally spaced around a round table. We have the job of assigning seats to the twelve participants. In how many different ways can we make these assignments so that each French farmer has an English farmer sitting immediately to her right.

5. Find the value $f(2015)$, if we know that the function f is defined on positive integers and satisfies

$$f(n) = \begin{cases} n-10, & \text{if } n > 10000, \\ f(f(n+11)), & \text{if } n \leq 10000. \end{cases}$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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