WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2014-2015) **January 2015**

1. Suppose that $x > 0$. Show that

$$\frac{2015}{(x+1)(x+4030)} \leq \frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{x+3} - \frac{1}{x+4} + - \cdots + \frac{1}{x+4029} - \frac{1}{x+4030}.$$

2. We have a cubic box whose interior dimensions are $6 \times 6 \times 6$. Clearly, we can fill it completely with 216 unit cubes. Is there an arrangement of 27 rectangular blocks with dimensions $1 \times 2 \times 4$ that exactly fills the cubic box? You can assume that in such an arrangement each rectangular block would cover exactly eight of the smaller unit cubes.

3. We choose three points in a square of unit side length and measure the distances between each pair of the points. Find the maximal possible value of the *minimum* of these three numbers.

4. Find all the four-digit numbers $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}$ which when multiplied by 4 give a product equal to the number with the digits reversed, $\underline{d}\,\underline{c}\,\underline{b}\,\underline{a}$. (The digits do not need to be different.)

5. In the sequence $2, 0, 1, 5, 8, \ldots$ each number is the last digit of the sum of the previous four. Show that the digits 2, 0, 1, 5 will show up eventually again in this order, but the digits 2, 0, 1, 4 will never show up in this order.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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