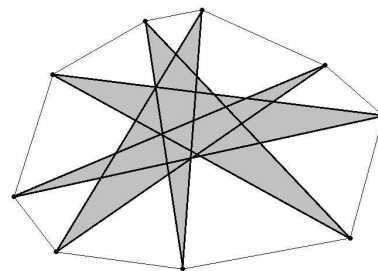


WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2014-2015)

December 2014

1. Find all integers a such that $a^3 + a + 1$ divides $a^4 + a + 1$.
2. You have 5 green and 7 red balls, and two empty boxes. You place all the balls in the two boxes so that each box contains at least one ball. Your friend then chooses one of the two boxes randomly, and picks a ball randomly from the chosen box. If the chosen ball is green, then you win a prize. How should you arrange the balls between the two boxes initially to maximize the probability of winning? What is this probability?
3. Consider the sequence a_0, a_1, a_2, \dots where $a_0 = 1$, and where $a_{k+1} = a_k + \frac{2}{a_k}$ for $k \geq 0$. (Thus $a_1 = 1 + \frac{2}{1} = 3$ and $a_2 = 3 + \frac{2}{3} = \frac{11}{3}$.) Prove that $a_{2015} > 89$.
4. Alice and Brittany play the following game. They start out with n marbles and take turns with Alice going first. In each turn Alice can take 1 or 2 marbles and Brittany can take 1, 3 or 5 marbles (if there are enough marbles left). The winner is the player who takes the last marble. For which n will Alice have a winning strategy?
5. Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$, and A_9 be the nine vertices of any convex nonagon in that order. Draw the 9 diagonals that connect each vertex to the vertex four away from it, that is, $A_1A_5, A_2A_6, A_3A_7, A_4A_8, A_5A_9, A_6A_1, A_7A_2, A_8A_3$, and A_9A_4 . These 9 diagonals form a nine-pointed star. (See the diagram for an example.) Show that the degree measures of the nine angles at the points of the star add to 180° .



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and other information about the talent search on our webpage:

<http://www.math.wisc.edu/talent>

| | | | |
|-----------------------|---|---------------------------------------|--------------|
| Return To | MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu | Deadline January 5, 2015 | |
| Or Email To | | Problem | Score |
| Please Fill In | PROBLEM SET II | 1 | |
| Name & Grade | | 2 | |
| School & Town | | 3 | |
| Home Address | | 4 | |
| Town & Zip | | 5 | |
| Email Address | | | |
| Teacher's Name | | | |