WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2014-2015) **November 2014**

1. Suppose that x and y are positive integers with $x+y = 10^{100}$. Assume that x and y have the exact same decimal digits, only in a different order. Show that both x and y are divisible by 5.

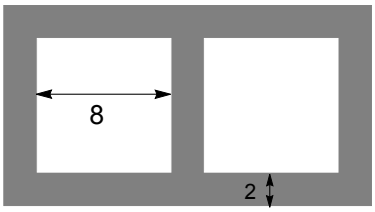
2. In the sequence $2, 3, 4, 6, 9, 14, \ldots$ each number is 1 less than the sum of the previous two terms. Show that all integers larger than 1 can be written as the sum of one or more distinct elements of the series.

3. The incircle of $\triangle ABC$ touches the sides of the triangle in three points. Show that if the triangle determined by the three touchpoints is similar to $\triangle ABC$, then it must be equilateral.

4. Show that if x, y, z are positive numbers, then

$$x^{2014}(x-y)(x-z) + y^{2014}(y-x)(y-z) + z^{2014}(z-x)(z-y) \geq 0$$

5. The diagram shows a 12×22 rectangular region with two 8×8 squares removed so that there is a border of width 2 around and between the squares. We would like to cut up the shaded region into non-overlapping triangles. Show that this is possible with 10 triangles, but impossible with 9 triangles.



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