WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2013-2014) **February 2014**

1. In the coordinate plane, we have points $C_1 = (0, 0)$, $C_2 = (18, 24)$, and $C_3 = (50, 0)$. Points A and B are in the plane so that the triangles $\triangle ABC_1$, $\triangle ABC_2$, and $\triangle ABC_3$ all have area 1200. Find all the possible lengths for the segment $\overline{AB}$.

2. Find all functions f from the positive real numbers to the real numbers such that for all positive real numbers x and y we have $f(x + y) = f(xy)$.

3. We have a 2014×2014 board made up of 2014^2 unit squares. We would like to cover the board in a single layer with 3×3 and 4×4 tiles (made up of 9 and 16 unit squares, respectively). Each used 3×3 (or 4×4) tile will have to cover exactly 9 (or 16) of the board's unit squares, and each unit square will have to be covered by exactly one of the used tiles. Is it possible to find such a tiling?

4. Let $X_1, X_2, \ldots, X_6$ be the vertices of a hexagon. Suppose that O is a point in the hexagon such that for all $1 \le i \le 6$ we have $\angle X_i O X_{i+1} = 60^\circ$ (where $X_7 = X_1$). Suppose $OX_1 < OX_3 < OX_5$ and $OX_2 < OX_4 < OX_6$. Show that

$$X_1X_2 + X_3X_4 + X_5X_6 < X_2X_3 + X_4X_5 + X_6X_1.$$

5. Is it possible to divide the numbers $1, 2, \ldots, 1000$ into two groups so that the sum of the squares of one group is equal to the sum of the squares of the other?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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