

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
PROBLEM SET IV (2013-2014) **January 2014**

1. Let ABC be a triangle with $AB = AC$, circumcenter O and incenter I . Let D be the point on AC such that line OD is perpendicular to line CI . Prove that ID and AB are parallel.
2. Show that if the sum of the fifth powers of five integers is divisible by 25 then one of the original integers is divisible by five.
3. We have an urn with 1000 balls numbered from 1 to 1000. We choose 9 balls randomly from the urn (without replacement) and add the shown numbers. Determine the probability that the sum is even.
4. Ann and Beth play the following game. They start out with n dimes on a table and take turns with Ann starting. In each step a player can take at most $n/2 + 1$ dimes from the table, but she has to take at least one. If somebody takes all the dimes on the table then she wins. For which values of n will Ann have a winning strategy?
5. The numbers from 1 to 100 are written in order around a circle. On each move Alice chooses an even number, y , on the circle, erases it along with its two neighbors, x and z , and replaces the three numbers with the sum of the two neighbors, $x + z$. She continues to make moves until either all the remaining numbers are odd or there are fewer than 3 numbers remaining. Prove that no matter how she chooses her moves, she will end up with 2 even numbers.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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