

**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
 PROBLEM SET III (2013-2014)**

December 2013

1. Somebody chose three points inside a circle of radius 1. Show that we can always choose two of the three points so that they are no more than distance $7/4$ apart. (Make sure to give complete justifications in your argument.)
2. Find all ordered triples (a, b, c) that satisfy the following system of equations:

$$ab + c = 6, \quad bc + a = 6, \quad ca + b = 6.$$
3. We have a special billiards table in the shape of a convex quadrilateral $ABCD$. A billiard ball starts at the midpoint of side AB and is hit so that it bounces off side BC at its midpoint, side CD at its midpoint, side DA at its midpoint, and returns to the position on side AB where it started. Find all shapes of quadrilateral $ABCD$ that would allow this to happen. (When the ball bounces off of one of the sides, the angle measured between the incoming path and the side of the table is always the same as the angle measured between the outgoing path and the side of the table.)
4. We have an urn with 3 red and k white balls. Let p be the probability that if we pick two balls randomly with replacement (i.e. we replace the first ball before choosing the second), then both balls are red. Let q be the probability that if we pick three balls randomly without replacement (i.e. we keep choosing from the remaining balls in each step), then all three balls are red. Find the value of k if we know that $p = 5q$.
5. Consider the number $n = 2014^{2014^{2014}}$. What is the first decimal digit after the decimal point in the number $\sqrt{n^2 + n + 1}$?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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