

**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH**

**PROBLEM SET II (2013-2014)**

**November 2013**

1. We wrote integer numbers on the vertices of a tetrahedron (one for each vertex). Show that if the sum of the numbers on each face is divisible by five, then all the numbers are divisible by five.
2. Let  $ABCD$  be a quadrilateral such that  $\angle DAB + \angle ABC = 120^\circ$ . Construct equilateral triangles  $ACE, BDF, CDG$ , with  $E, F, G$  each on the other side of  $AC, BD, CD$  (respectively) from  $AB$ . Show that  $E, F, G$  are collinear (i.e. there is a line that goes through all three).
3. Let  $n > 1$  be an integer and suppose that the polynomial  $p(x)$  has degree  $n$  and satisfies  $p(1) = 3, p(2) = 5, p(3) = 7, \dots, p(n) = 2n + 1$ , and  $p(n + 1) = 2n + 5$ . Evaluate  $p(n + 3)$ .
4. Is there a function  $f(x)$  defined on all real numbers  $x$  so that  $f(f(x)) = -x$  for all  $x$ ? (Either prove that there is no such  $f$ , or give an example of one.)
5. 2013 students each roll 9 standard six-sided dice and record how many times each of the numbers 1, 2, 3, 4, 5, 6 appear. Show that there are at least two students who recorded the same result.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and other information about the talent search on our webpage:

<http://www.math.wisc.edu/talent>

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	<b>Deadline</b> December 2, 2013	
<b>Or Email To</b>	talent@math.wisc.edu		
<b>Please Fill In</b>	<b>PROBLEM SET II</b>	<b>Problem</b>	<b>Score</b>
Name & Grade		1	
School & Town		2	
Home Address		3	
Town & Zip		4	
Email Address		5	
Teacher's Name			

Please check here if you would like to be emailed when new problem sets are available.