1. There are two non-intersecting circles $C_1$ and $C_2$, with centers $X_1$ and $X_2$ respectively, with neither circle inside the other. From $X_1$ and $X_2$, we draw the tangents to the opposite circles. For $i = 1, 2$, the tangents from $X_i$ intersect $C_i$ at points $A_i$ and $B_i$. (We label the points so that $A_1$ and $A_2$ are on the same side of the line $X_1 X_2$.) Show that the line segments $A_1 B_1$ and $A_2 B_2$ have equal length.

2. Suppose that $a \geq b \geq c \geq 0$ and $a + b + c \leq 1$. Show that $a^2 + 3b^2 + 5c^2 \leq 1$.

3. We would like to find sets $A_1, A_2, \ldots, A_n$ of size three which are all subsets of $\{1, 2, \ldots, 100\}$ and for any $1 \leq a < b \leq 100$ there is exactly one $A_i$ with $\{a, b\} \subset A_i$. Decide if it is possible to construct such sets.

4. Given a set of $2n + 1$ points on a circle, prove that there are at most $\frac{1}{6} n(n + 1)(2n + 1)$ acute triangles with vertices at those points.

5. In a school we have $n$ girl and $n$ boy students with $n > 2013$. We know that the number of ways we can choose a club consisting of 5 boys and 6 girls is a square number. What’s the smallest possible value of $n$?

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You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.