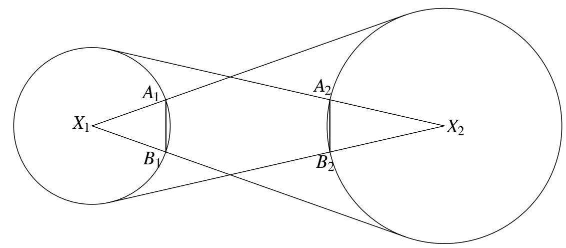
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2012-2013) **February 2013**

1. There are two non-intersecting circles C_1 and C_2, with centers X_1 and X_2 respectively, with neither circle inside the other. From X_1 and X_2, we draw the tangents to the opposite circles. For $i = 1, 2$, the tangents from X_i intersect C_i at points A_i and B_i. (We label the points so that A_1 and A_2 are on the same side of the line X_1X_2.) Show that the line segments A_1B_1 and A_2B_2 have equal length.



2. Suppose that $a \geq b \geq c \geq 0$ and $a + b + c \leq 1$. Show that $a^2 + 3b^2 + 5c^2 \leq 1$.

3. We would like to find sets $A_1, A_2, \ldots, A_n$ of size three which are all subsets of $\{1, 2, \ldots, 100\}$ and for any $1 \leq a < b \leq 100$ there is exactly one A_i with $\{a, b\} \subset A_i$. Decide if it is possible to construct such sets.

4. Given a set of $2n+1$ points on a circle, prove that there are at most $\frac{1}{6}n(n+1)(2n+1)$ acute triangles with vertices at those points.

5. In a school we have n girl and n boy students with $n > 2013$. We know that the number of ways we can choose a club consisting of 5 boys and 6 girls is a square number. What's the smallest possible value of n?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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Problem	**Score**
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