WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2012-2013) **November 2012**

1. Prove that a prime number cannot be expressed as the sum of two or more consecutive positive odd integers.

2. From a 29×29 grid of unit squares we cut out ninety-nine 2×2 squares consisting the squares of the grid. Show that we can cut out one more!

3. A semicircle has a diameter XY on which points M and N lie. The semicircle contains points A, B, C, D such that $\angle AMX = \angle CMY = \angle BNX = \angle DNY$. Prove that $AC = BD$.

4. We have an infinite sequence of numbers $f_1, f_2, f_3, \ldots$ which satisfy

$$f_{\frac{x+y}{3}} = \frac{f_x + f_y}{2}$$

whenever x, y and $\frac{x+y}{3}$ are all positive integers. (f_n denotes the element of the sequence at position n.) How many distinct values can appear in the sequence?

5. Show that

$$3 - \frac{1}{5^{2011}} < \underbrace{\sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots + \sqrt{6 + \sqrt{6}}}}}}_{\text{2012 square roots}} < 3.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Return To **Or Email To**	**MATHEMATICS TALENT SEARCH** Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu		**Deadline** December 1, 2012	
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Name & Grade			1	
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