1. Prove that a prime number cannot be expressed as the sum of two or more consecutive positive odd integers.

2. From a $29 \times 29$ grid of unit squares we cut out ninety-nine $2 \times 2$ squares consisting the squares of the grid. Show that we can cut out one more!

3. A semicircle has a diameter $XY$ on which points $M$ and $N$ lie. The semicircle contains points $A, B, C, D$ such that $\angle AMX = \angle CMY = \angle BNX = \angle DNY$. Prove that $AC = BD$.

4. We have an infinite sequence of numbers $f_1, f_2, f_3, \ldots$ which satisfy

$$f_{\frac{x+y}{2}} = \frac{f_x + f_y}{2}$$

whenever $x, y$ and $\frac{x+y}{2}$ are all positive integers. ($f_n$ denotes the element of the sequence at position $n$.) How many distinct values can appear in the sequence?

5. Show that

$$3 - \frac{1}{5^{2011}} < \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots + \sqrt{6 + \sqrt{6}}}}} < 3.$$ 

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.