1. Find all positive integers $n$ such that $n^3$ and $n^4$ contain, between them, each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 exactly once.

2. Let $ABC$ be an acute triangle with median $BD$ and altitude $CE$, where $BD = CE$ and $\angle DBC = \angle ECB$. Show that $ABC$ is equilateral.

3. We color each point of a unit cube with one of three colors. Is it true that there are necessarily two points in the cube with the same color with distance at least 1.4? How about 1.5?

4. Let $m, n$ be integers such that $23^{2011}$ divides $m^2 + n^2$. Show that $23^{2012}$ divides $mn$.

5. Consider 16 lattice points arranged on a $4 \times 4$ square grid. We color the first point of the third row black and the other 15 points with white. Next in each step we can choose a horizontal or a vertical line or a line which is parallel to one of the main diagonals and we can change the colors of the lattice points on that line to the opposite. Is it possible to change the colors of all the lattice points to white?