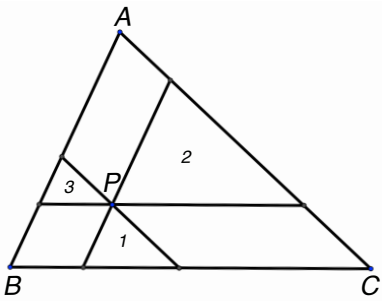
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2011-2012) **FEBRUARY 2012**

1. How many dice should we roll if we want to make the probability of having exactly two sixes as large as possible?

2. In the figure, point P lies in the interior of $\triangle ABC$, and lines parallel to the sides of the triangle have been drawn through P. These lines decompose the original triangle into three quadrilaterals and three triangles, and we label the three small triangles 1, 2 and 3, as shown. Let a be the area of $\triangle ABC$, and write a_1, a_2 and a_3 to denote the areas of triangles 1, 2 and 3, respectively. Prove that $\sqrt{a} = \sqrt{a_1} + \sqrt{a_2} + \sqrt{a_3}$.



3. We have 100 piles of coins, with the i^{th} pile containing exactly i coins. We wish to remove all the coins in a series of steps. In each step we are allowed to take away coins from as many piles as we wish, but we have to take the same number of coins from each such pile. Show that we can remove all the coins in 7 steps, but that it is impossible to do so in 6.

4. Find all integers n such that $n^2 - 13n + 50$ is a perfect square, and prove that you have found them all.

5. Show that if a and b are real numbers bigger than $\frac{1}{2}$ then

$$a + 2b - 5ab < \frac{1}{4}.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Problem	**Score**
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