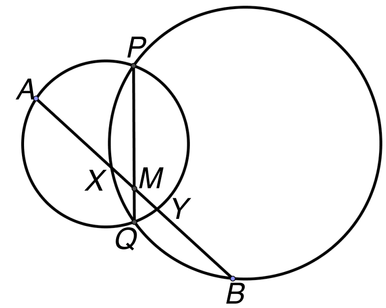
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2011-2012) **JANUARY 2012**

1. Suppose that a rectangular array of numbers has the following two properties. First, there is some constant A such that in every row, the sum of the largest and smallest numbers is A, and second, there is some constant B such that in every column, the sum of the largest and smallest numbers is B. Prove that $A = B$.

2. In the figure, line $\overline{AB}$ joins point A on one circle to point B on another circle, and X and Y are the other two points where $\overline{AB}$ meets the circles, as shown. Assume that the common chord $\overline{PQ}$ passes through the midpoint M of $\overline{AB}$. Prove that M is also the midpoint of $\overline{XY}$.



3. In a group of 30 students, each student knows exactly six of the remaining 29 students. Of course, A knows B if and only if B knows A. We will call a group of <u>three</u> students *balanced* if either all three students know each other or no one knows anyone else within the group. In how many ways can we choose a balanced group?

4. Find the maximum of the expression

$$x^4y + x^3y + x^2y + xy + xy^2 + xy^3 + xy^4$$

if x, y are real numbers with $x + y = 2$.

5. (New Year's Problem) Show that there exist integers X and Y such that the product

$$P = (1^2 + 2^2)(2^2 + 3^2)(3^2 + 4^2) \cdots (2011^2 + 2012^2)$$

is equal to $X^2 + Y^2$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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