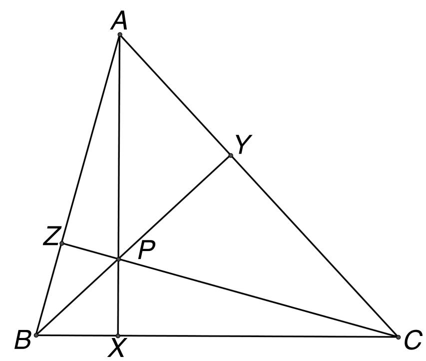
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2011-2012) **DECEMBER 2011**

1. We place 12 coins on the squares of an 8×8 chessboard (we might put more than one coin on a given square). Show that we can erase four rows and four columns from the board so that the remaining 16 squares do not contain any coins.



2. In the figure at the right, lines $\overline{AX}$, $\overline{BY}$ and $\overline{CZ}$ have been drawn in $\triangle ABC$, and point P lies on each of these three lines. Also, $\angle XAY = \angle XBY$ and $\angle XAZ = \angle XCZ$. Prove that $\angle ZBY = \angle ZCY$.

3. How many solutions, in positive integers x and y, are there for the equation $x^2 = y^2 + 4{\cdot}3^{2011}$.

4. Let a and $x_1, x_2, \ldots, x_n$ be positive numbers, and suppose that the sum of the x_i exceeds a. For each subset J of $\{1, 2, \ldots, n\}$, write s_J to denote the sum of those numbers x_j for which j is in J, and note that $s_J = 0$ if J is the empty set. Assume also that whenever $s_J < a$, we have $x_i \le a - s_J$ for all subscripts i not in J. Show that all x_i are equal and that a is an integer multiple of their common value.

5. If x, y and z are positive numbers, prove that

$$3\Big(\frac{x+y}{z} + \frac{y+z}{x} + \frac{z+x}{y}\Big) \ge 2(x+y+z)\Big(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\Big)\,.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Deadline	
January 2, 2012	
Problem	**Score**
1	
2	
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