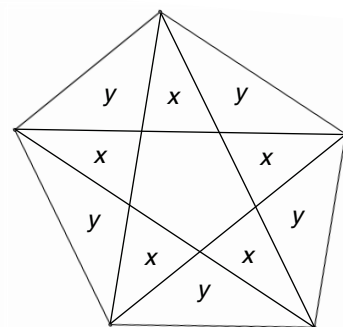


1. We are given 13 (positive) integer weights and we know that if we discard any one of them then we can divide the rest into two groups of six so that the total weights are the same in the two groups. Show that all the weights must be equal.



2. A pentagon is divided into a smaller pentagon and ten triangles, as in the diagram. Of the ten triangles, five have area  $x$  and five have area  $y$ , as indicated. Compute the ratio  $y/x$ .

3. Suppose that  $f(x)$  is a function that is defined for all rational numbers  $x$ , and assume that  $f(x) = f(x - 2)$  for all such  $x$ . Assume also that if  $x$  is nonzero, then  $f(x) = f(1/x)$ . Show that for every rational  $x$ , either  $f(x) = f(0)$  or  $f(x) = f(1)$ .

4. Suppose we have two urns, each containing some red and some blue marbles, with at least one of each color in each urn. Assume that if we choose an urn randomly and then choose a marble randomly from that urn, then the probability of picking a red marble is the same as we would get by combining all the marbles into one urn and choosing a marble from that one at random. If the first urn contains 7 marbles and the second one contains 5 red marbles, how many marbles can there be in the second urn?

5. Find all prime numbers  $p < q$  such that  $p^2q^2 + 5$  is also prime.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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