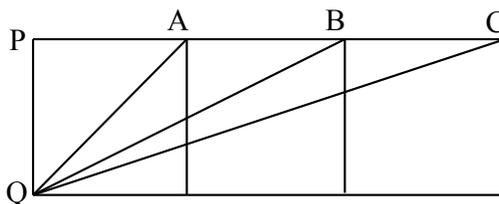


WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH  
 PROBLEM SET I (2011-2012) OCTOBER 2011

1. Show that if we have 100 distinct points on the plane, then we can find a line on the plane so that exactly 50 of the 100 points lie on each side of the line.
2. The picture below shows three adjacent unit squares. Determine the sum of the three angles  $\angle PAQ$ ,  $\angle PBQ$  and  $\angle PCQ$ .



3. Suppose that  $x$ ,  $y$  and  $z$  are integers with the property that no integer larger than 1 exactly divides all of them. Let

$$A = x + 2y, \quad B = y + 2z, \quad \text{and} \quad C = z + 2x.$$

Find the largest integer that can exactly divide all of  $A$ ,  $B$  and  $C$ .

4. We can think of a domino as two unit squares side-by-side. Similarly, a tetromino is four unit squares joined at their sides. Tetrominoes come in several different shapes, but for this problem, we consider only “L-tetrominoes”, shaped either like the letter “L” or its mirror image, with three squares in a row and the fourth attached at the side at one end of the row. Decide whether or not it is possible to cover a  $10 \times 10$  square with nonoverlapping L-tetrominoes, and prove that your answer is correct.
5. Let  $a$  and  $b$  be positive integers. Find all solutions to the equation  $a^b + b^a = 2ab$ .

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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