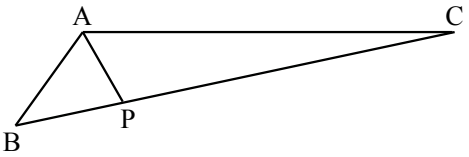
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2010-2011) **FEBRUARY 2011**

1. Find the largest positive integer n such that n is equal to the cube of the sum of its digits.

2. In $\triangle ABC$, angle A is 120° and the lengths of sides $\overline{AB}$ and $\overline{AC}$ are 4 and 12, respectively. If $\overline{AP}$ bisects angle A, where P lies on $\overline{BC}$, compute the length AP (with proof, of course).



3. The integers from 0 to 100 are written in order, horizontally across a blackboard, with spaces between the numbers. Alice puts 50 plus signs and 50 stars in some random arrangement, into the 100 spaces between the numbers. Then (interpreting "$*$" as multiplication) she computes the value of what is written, obtaining the answer a. (Of course, Alice follows the usual rule: multiplication before addition. Thus, for example, the expression $0+1*2+3*4$ has the value 14.) Bob changes all of Alice's pluses to stars and all of her stars to pluses, and then he computes the value of the new expression, obtaining the answer b. If $a+b$ is a number whose rightmost four digits are 2011, prove that at least one of Alice or Bob made an arithmetic mistake.

4. Let x, y and z be nonzero real numbers, and suppose that $x+y+z=3$ and $xy+yz+zx=0$. Let $p=xyz$. Prove that $-4 \le p < 0$ and find all possibilities for x, y and z such that $p=-4$.

5. In a certain country, a dollar is 100 cents and coins have denominations 1, 2, 5, 10, 20, 50 and 100 cents. Suppose that one can make A cents using exactly B coins. Prove that it is possible to make B dollars using exactly A coins.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Problem	**Score**
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