

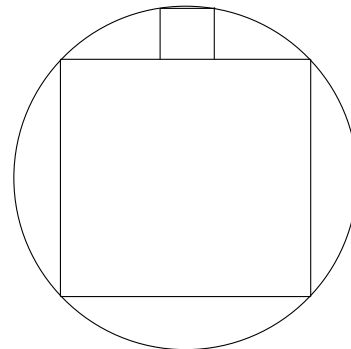
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2010-2011)

JANUARY 2011

1. Suppose that for each integer x , a certain integer x^* is defined, where x^* depends on x . Assume that there are only finitely many different numbers x^* as x runs over the set of all integers, and assume also that when x and y are different integers, it is never the case that $x + kx^* = y + ky^*$, for any integer k . Show that there exists a positive integer m such that $(x + m)^* = x^*$ for all integers x .

2. In the diagram, a square is inscribed in a circle, and a smaller square is drawn with one side along a side of the large square and two vertices on the circle. What fraction of the side length of the large square is the side length of the small square?



3. Find all solutions of the simultaneous equations

$$2x^2 = 14 + yz \quad 2y^2 = 14 + zx \quad 2z^2 = 14 + xy.$$

4. Suppose that n is a positive integer that is not a power of 2. Show that n can be written as a sum of at least two consecutive integers.

5. I know that the population of a certain village consists of 100 humans and 15 extraterrestrial aliens. When I visit the village, I am unable to distinguish the humans from the aliens, but I want to find someone who I am sure is human. I ask each villager to give me the names of some human residents, allowing self-nominations. Everyone submits a list of 15 names, but while the humans' lists contain only the names of humans, I have no such assurance about the aliens' lists. Show that I can select a human when I examine the 115 unsigned lists.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	Deadline February 1, 2011	
Or Email To		Problem	Score
Please Fill In	PROBLEM SET IV		
Name & Grade		1	
School & Town		2	
Home Address		3	
Town & Zip		4	
Email Address		5	