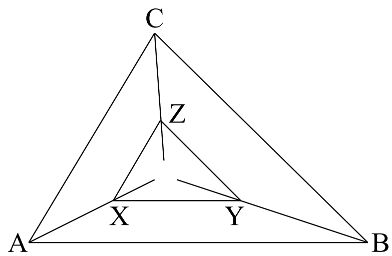
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2010-2011) **DECEMBER 2010**

1. I have five clocks, each of which chimes when it shows a full hour. These clocks run at constant, but possibly different and incorrect rates, but nevertheless, I notice that at each hour (according to my accurate wristwatch) at least two of my clocks chime. Prove that we can throw away at least three of the five clocks and still hear a chime at each true hour.

2. In the figure, $\triangle XYZ$ is inside $\triangle ABC$, and sides $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ are parallel to sides $\overline{XY}$, $\overline{XZ}$ and $\overline{YZ}$, respectively. Prove that lines $\overline{AX}$, $\overline{BY}$ and $\overline{CZ}$ go through a common point.



3. Let $a < b$ be positive integers, where $a + b$ is odd, and let k be any odd positive integer. Show that the number

$$a^k + (a+1)^k + (a+2)^k + \cdots + (b-1)^k + b^k$$

is a multiple of $a + b$.

4. (New Year's Problem.) Suppose that $\square$ is an operation defined on the integers, and assume that the following conditions are satisfied.

(a) $(x + y) \square z = y \square (x + z)$ for all x, y and z.

(b) $(3x) \square y = x \square (3y)$ for all x and y.

(c) $1 \square 1 = 2011$.

Compute $2011 \square 2011$.

5. Prove that

$$2^{\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}\right)} < n$$

for all integers $n \geq 2$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Deadline	
January 3, 2011	
Problem	**Score**
1	
2	
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