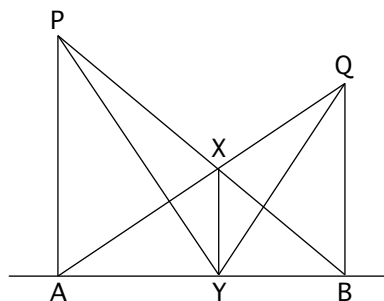


- Twenty lines are drawn in the plane with no pair of them parallel. Every two of the lines thus intersect at a point, and we suppose that there are exactly 174 points that lie on more than one of our lines. Prove that some point lies on exactly three of the given lines.



- In the diagram, lines  $\overline{PA}$  and  $\overline{QB}$  are perpendicular to line  $\overline{AB}$ . Point  $X$  is the intersection of  $\overline{PB}$  and  $\overline{QA}$ , and the perpendicular  $\overline{XY}$  is dropped from  $X$  to  $\overline{AB}$ . Prove that  $\overline{XY}$  bisects  $\angle PYQ$ .

- Let  $f$  and  $g$  be real-valued functions defined on the real numbers, and let the sets  $A$  and  $B$  be given by  $A = \{x \mid f(g(x)) = x\}$  and  $B = \{x \mid g(f(x)) = x\}$ . Assuming that  $A$  and  $B$  are finite (but possibly empty) sets, show that  $A$  and  $B$  contain equal numbers of elements.
- Find all integers  $n$  with the property that

$$\sqrt{n + 12\sqrt{5}} - \sqrt{n - 12\sqrt{5}}$$

is also an integer.

- At noon, the hour hand, minute hand and second hand of an accurately set clock all point in the same direction. Find the next time that the three hands coincide, and prove that your answer is correct.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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