

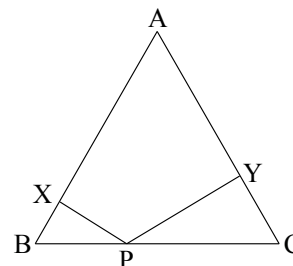
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2010-2011)

OCTOBER 2010

1. Find all positive integer solutions of  $x^x + y^y + z^z = 3xyz$ .

2. Suppose that  $\triangle ABC$  is isosceles with base  $\overline{BC}$ . A point  $P$  on  $\overline{BC}$  is chosen, and perpendiculars  $\overline{PX}$  and  $\overline{PY}$  are dropped from  $P$  to  $\overline{AB}$  and  $\overline{AC}$  (possibly extended). Show that the total length  $PX + PY$  remains constant as  $P$  moves along side  $\overline{BC}$ .



3. Twenty closed fruit crates are lined up on the floor. Ten of the crates are labeled “APPLES” and the other ten are labeled “PEARS”. I want to select at least one crate of apples and one of pears from among the crates. This would be easy, except that some of the crates have been incorrectly labeled, and I do not know which are the mislabeled crates or how many of them there are. I do know, however, that of every five consecutive crates, at most two are wrongly labeled. Find a strategy that allows me to choose six crates in such a way that I can be sure that at least one crate of apples and one of pears is among my six. Also, prove that there is no strategy guaranteed to accomplish this when only five crates are selected.

4. I have 64 numbered tiles which contain 64 different numbers. I want to place these tiles into the 64 boxes of an  $8 \times 8$  chessboard. There are of course, a huge number of ways to do this. Let us say that a placement of tiles into boxes is “good” if the sums of the tiles in all eight rows are equal, and that the placement is “very bad” if no two of the row sums are equal. Prove that there cannot be more good placements than there are very bad placements.

5. Let  $a > 0$  and  $b > 0$ . Prove that

$$\frac{a}{a + b^4} + \frac{b}{b + a^4} > \frac{1}{1 + a^2b^2}.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	<b>Deadline</b> November 1, 2010	
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