1. Recall that most cubic polynomials have either three distinct real roots or just one such root. Sometimes, however, a cubic has exactly two distinct real roots because two of the three roots coincide. Find all real numbers \( a \) such that the polynomial \( x^3 - 7x^2 + ax - 9 \) has exactly two distinct real roots.

2. In the diagram, \( \angle BAC = 90^\circ \). The bisector of this angle is extended to meet the circumcircle of \( \triangle ABC \) at point \( P \). Show that \( AB + AC = \sqrt{2} AP \).

3. Suppose that 100 on-off light switches are mounted on a control panel in one long line, and that I can flip any batch of consecutive switches simultaneously. Some of the lights are on and some are off, and I want to turn them all off. For example, if lights 1 through 30 are on; lights 31 through 60 are off and lights 61 through 100 are on, I can turn all the lights off in two moves: first flip switches 1 through 30 and then flip switches 61 through 100. Find the smallest number \( N \) so that I never need more than \( N \) moves to turn off all the lights.

4. Find all solutions in real numbers \( x, y \) and \( z \) for the equations
   \[
   x^2 + y^2 + z^2 = xy + xz + yz = xyz.
   \]

5. Does there exist a positive integer \( n \) for which it is possible to write \( 1/n = 1/a^2 + 1/b^2 \), where \( a \) and \( b \) are unequal positive integers? If so, find the smallest such number \( n \).

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.