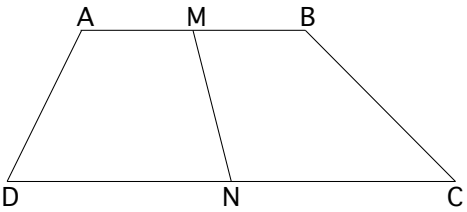
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2009-2010) **NOVEMBER 2009**

1. Show that the equation $a^2+b^2 = c^2+5$ has infinitely many different positive integer solutions.

2. In the figure, $ABCD$ is a quadrilateral, M and N are the midpoints of sides $\overline{AB}$ and $\overline{CD}$, respectively, and line $\overline{MN}$ cuts the quadrilateral into two parts with equal areas. Show that sides $\overline{AB}$ and $\overline{CD}$ are parallel.



3. Black and White are playing the following game using the number line as a game board. First, Black places some markers on positive integer points. Then White starts at the origin and repeatedly jumps to the right, always landing at an integer, and always moving at least 1 unit. The rules specify that White's first jump is at most 100 units, and that each later jump is no longer than the previous one. (Thus, for example, White's first four jumps might be 100, 95, 95 and 30 units.) Black wins if White lands on one of Black's markers, and White wins if he can get beyond all of the markers without landing on any of them. Find the smallest number of Black markers that will guarantee a Black win.

4. Let x, y and z be integers, and consider the quantity $Q = 16(x^2 + y^2 + z^2) - 5(x + y + z)^2$. Prove that $Q \geq 0$, and find the smallest positive number that Q can be.

5. Let

$$S_n = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{4n+1} - \frac{1}{4n+3}\,.$$

Show that $S_n < 5/6$ for every integer $n \geq 0$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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