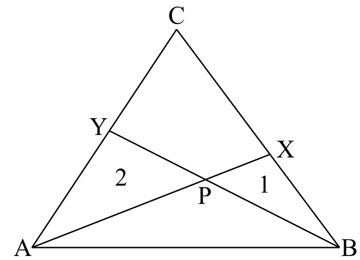
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2008-2009) **JANUARY 2009**

1. (**New Year's Problem**.) Find all positive integers n such that $n^2 + n + 2009$ is a square.

2. In the figure, the area of $\triangle ABC$ is a whole number. Lines $\overline{AX}$ and $\overline{BY}$ are drawn, where X lies on side $\overline{BC}$ and Y lies on side $\overline{AC}$, and these lines meet at point P, inside the triangle. The area of $\triangle BPX$ is 1, the area of $\triangle APY$ is 2, and the area of $\triangle APB$ is a whole number. Find the area of $\triangle ABC$, and prove that your answer is correct.



3. Find a simple expression (in terms of n) for the sum S_n of all of the numbers of the form $k2^k$ where k is an integer and $1 \leq k \leq n$.

4. Decide (with proof) whether or not there exists a set $\mathcal{E}$ of even positive integers such that every even positive integer can be written in a unique way as a sum of distinct members of $\mathcal{E}$. Similarly, decide if there exists a set $\mathcal{O}$ of odd positive integers such that every odd positive integer can be written in a unique way as a sum of distinct members of $\mathcal{O}$.

5. Let n be a positive integer. A deck of $2n$ numbered cards contains exactly two cards marked with each of the integers from 1 to n, and these are arranged in the order $1, 1, 2, 2, 3, 3, \ldots, n, n$ from top to bottom. Observe that if $n = 3$, the deck can be cut into two pieces, namely $1, 1, 2, 2$ and $3, 3$, so that the sums of the numbers on the cards in the top and bottom parts are equal. Prove that there are infinitely many positive integers n for which the deck *cannot* be cut into two pieces so that the sums of the cards in the top and bottom parts are equal.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To **Or Email To**	**MATHEMATICS TALENT SEARCH** Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	**Deadline** February 2, 2009
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1	
2	
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