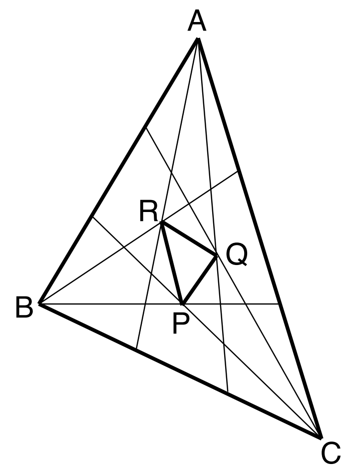
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2008-2009) **DECEMBER 2008**

1. A string of 0s and 1s is said to be *palindromic* if it reads the same backwards and forwards. Thus, for example, 101 and 1001 are palindromic, but 1101 is not. Now let S be an infinitely long string of 0s and 1s with no beginning and no end. Show that S contains a palindromic string consisting of four or more consecutive symbols. Decide whether or not S must contain a palindromic string consisting of five or more consecutive symbols.



2. In the diagram, each side of $\triangle ABC$ is trisected, and each vertex is joined by line segments to the two trisection points of the opposite side. Let P, Q and R be the intersections of three pairs of these line segments, as shown. Prove that $\triangle PQR$ is similar to $\triangle ABC$, and compute its area as a fraction of the area of the big triangle.

3. Find all positive integers n such that $(n^3 + 100)/(n^2 + 100)$ is an integer.

4. Let $\square$ be an operation defined on the integers. (In other words, given integers a and b, we get an integer $a \,\square\, b$ determined by a and b.) Assume the following three axioms.

(1) $(a + b)(a \,\square\, b) = (a^2) \,\square\, (b^2)$ for all integers a and b.

(2) $(a \,\square\, b) + (b \,\square\, c) = a \,\square\, c$ for all integers a, b and c.

(3) $1 \,\square\, 0 = 1$.

Show that $a \,\square\, b = a - b$ for all integers a and b.

5. Let n be a positive integer. Show that $K_n = 2^{2n-1} - 9n^2 + 21n - 14$ is a multiple of 27.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To **Or Email To**	**MATHEMATICS TALENT SEARCH** Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu
Please Fill In	**PROBLEM SET III**
Name & Grade	
School & Town	
Home Address	
Town & Zip	
Email Address	

Deadline January 5, 2009	
Problem	**Score**
1	
2	
3	
4	
5	