1. A string of 0s and 1s is said to be palindromic if it reads the same backwards and forwards. Thus, for example, 101 and 1001 are palindromic, but 1101 is not. Now let $S$ be an infinitely long string of 0s and 1s with no beginning and no end. Show that $S$ contains a palindromic string consisting of four or more consecutive symbols. Decide whether or not $S$ must contain a palindromic string consisting of five or more consecutive symbols.

2. In the diagram, each side of $\triangle ABC$ is trisected, and each vertex is joined by line segments to the two trisection points of the opposite side. Let $P$, $Q$ and $R$ be the intersections of three pairs of these line segments, as shown. Prove that $\triangle PQR$ is similar to $\triangle ABC$, and compute its area as a fraction of the area of the big triangle.

3. Find all positive integers $n$ such that $(n^3 + 100)/(n^2 + 100)$ is an integer.

4. Let $\boxplus$ be an operation defined on the integers. (In other words, given integers $a$ and $b$, we get an integer $a \boxplus b$ determined by $a$ and $b$.) Assume the following three axioms.

   (1) $(a + b)\boxplus (a \boxplus b) = (a^2) \boxplus (b^2)$ for all integers $a$ and $b$.

   (2) $(a \boxplus b) + (b \boxplus c) = a \boxplus c$ for all integers $a$, $b$ and $c$.

   (3) $1 \boxplus 0 = 1$.

   Show that $a \boxplus b = a - b$ for all integers $a$ and $b$.

5. Let $n$ be a positive integer. Show that $K_n = 2^{2n-1} - 9n^2 + 21n - 14$ is a multiple of 27.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.