

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

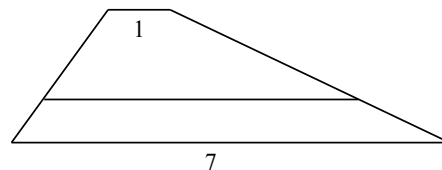
PROBLEM SET II (2008-2009)

NOVEMBER 2008

1. Let  $x, y$  and  $z$  be positive real numbers. Show that

$$(xy + yz + zx)(x + y + z) \geq 9xyz.$$

2. The parallel sides of a trapezoid have lengths 1 and 7, and the area of the trapezoid is divided into two equal parts by a line segment parallel to those two sides. Find the length of that line segment, and prove that your answer is correct.



3. Each member of a sports team is a friend of at least six others on the team, and yet there are no four members of the team such that each of them is a friend of the other three. Find (with proof, of course) the smallest size team for which this is possible. (You should assume that the “friend” relationship is symmetric. In other words, if A is a friend of B, then also, B is a friend of A.)
4. Note that  $654/545 = 6/5$ , so that one can “cancel” the 54 in the numerator and denominator of  $654/545$  without changing the value of the fraction. Now consider the fraction  $6545454 \cdots 54/545454 \cdots 545$ , where there are  $n$  copies of 54 following the digit 6 of the numerator, and the same number  $n$  of copies of 54 preceding the final digit 5 of the denominator. Prove that this fraction is equal to  $6/5$  for every positive integer  $n$ .
5. Recall that  $n!$  (pronounced “ $n$ -factorial”) is defined to be the product of all of the integers from 1 through  $n$ . Prove that  $n! \leq 2(n/2)^n$  for all positive integers  $n$ .

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	<b>Deadline</b> December 1, 2008	
<b>Or Email To</b>		<b>Problem</b>	<b>Score</b>
<b>Please Fill In</b>	<b>PROBLEM SET II</b>	1	
Name & Grade		2	
School & Town		3	
Home Address		4	
Town & Zip		5	
Email Address			