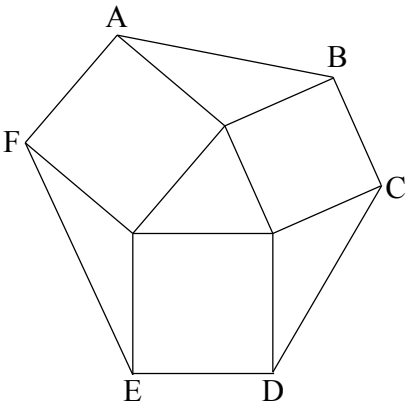
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2008-2009) **OCTOBER 2008**

1. Let $p = 101010\cdots01$ be the m-digit number in which the first and last digits are 1 and the digits alternate between 1 and 0. For which positive integers m is the number p prime.

2. In the figure on the right, hexagon $ABCDEF$ is divided into three squares and four triangles. Show that the areas of all four triangles are equal.



3. Find all positive integers x and y such that $x \le y \le 2x$ and $1 + x^2 + y^2 = 3xy$.

4. Your calculator will show that the number

$$\sqrt[3]{7+5\sqrt{2}} + \sqrt[3]{7-5\sqrt{2}}$$

is approximately an integer. Decide whether or not it is exactly an integer, and prove that your answer is correct.

5. Let F_n be the nth Fibonacci number. Thus $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, and in general for $n > 2$, we have $F_n = F_{n-1} + F_{n-2}$. Now for each integer $n \ge 1$, write $A_n = (F_{n+1})^2 - (F_n)^2 - F_nF_{n+1}$ and $B_n = (F_n)^2 + (F_{n+2})^2 - 3F_nF_{n+2}$. Find simple formulas for A_n and B_n and use them to compute A_{1000} and B_{1000}.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Deadline November 3, 2008	
Problem	**Score**
1	
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