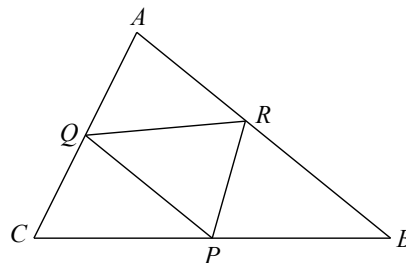


1. Fix a real number  $k > 0$ . Let  $x_1, x_2, \dots, x_n$  be  $n$  positive real numbers and suppose that  $x_1(2k - x_2) = k^2$ ,  $x_2(2k - x_3) = k^2$ ,  $x_3(2k - x_4) = k^2, \dots$ , and  $x_n(2k - x_1) = k^2$ . Show that all  $x_i$  are equal to  $k$ .

2. Given triangle  $\triangle ABC$ , let point  $P$  lie on side  $\overline{BC}$ , point  $Q$  on side  $\overline{CA}$  and point  $R$  on side  $\overline{AB}$ . Suppose that the four triangles  $\triangle ARQ$ ,  $\triangle BPR$ ,  $\triangle CQP$  and  $\triangle PQR$  all have equal area. Show that  $P$ ,  $Q$  and  $R$  are the midpoints of the respective sides.



3. Let  $d$  and  $n$  be positive integers, and let  $r$  be coprime to  $d$ . Show that for some integer  $k$ , the number  $r + kd$  is coprime to  $n$ .
4. We call the squares of a 5 by 5 checkerboard adjacent if they share a side. What is the minimum number of squares that one can color yellow on such a 5 by 5 board so that every non-yellow square is adjacent to a yellow square?
5. Do there exist polynomials  $P(x)$ ,  $Q(x)$  and  $R(x)$  having integer coefficients such that  $P(x)$  is the product  $P(x) = Q(x)R(x)$ , all coefficients of  $P(x)$  are 0, 1 or  $-1$ , and one of the coefficients of  $Q(x)$  is 2008?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

|                       |                                                                                                                                                   |                                     |              |
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