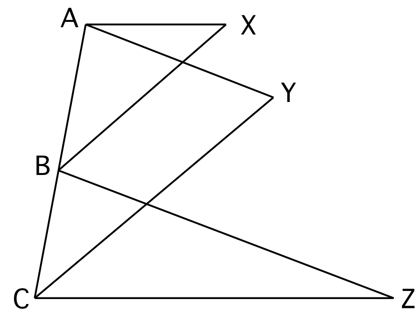
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2007-2008) **JANUARY 2008**



1. If p is a prime, find all positive integer solutions p, a, b, c to the equation

$$\frac{1}{p} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}.$$

2. Let A, B, C, X, Y, and Z be six distinct points in the plane with A, B, and C collinear. If $\overline{AX}$ is parallel to $\overline{CZ}$, $\overline{AY}$ is parallel to $\overline{BZ}$, and $\overline{BX}$ is parallel to $\overline{CY}$, prove that X, Y, and Z are collinear.

3. (New Year's Problem) Let S be the set of all positive integers s such that $2^{2008} + 2^s + 1$ is a square. Find the smallest member of S and prove that it is not the only member of S.

4. Find the functions $f(x)$ that satisfy

$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{1}{x(1-x)}$$

for all real numbers $x \neq 0$, 1.

5. Let n be a positive integer, and recall that the binomial coefficient

$$b(n) = \binom{2n}{n} = \frac{(2n)!}{n!\,n!}$$

is also an integer. Show that $b(n)$ is always even and that it is divisible by 4 unless n is a power of 2.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Deadline	
February 1, 2008	
Problem	**Score**
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