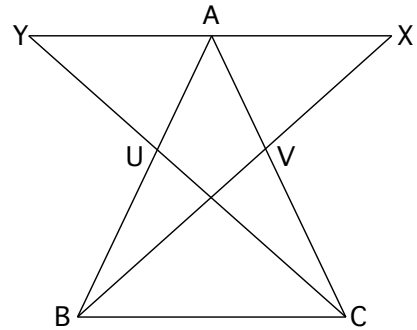


WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
PROBLEM SET III (2007-2008) **DECEMBER 2007**

1. Numerican University offers courses in 5 subjects, call them A, B, C, D, and E. Every student is interested in exactly one of these, but some subjects require taking courses in certain other subjects simultaneously. No student takes courses in subjects he is not interested in unless he is forced to do so. Currently, there are 200 students at NU and the enrollment in courses A, B, C, D, and E is 100, 110, 120, 130, and 150, respectively. If a student is interested in A, how many other courses must he take? What about B?



2. In the figure, $\triangle ABC$ is given, and line \overline{XY} is drawn through A, parallel to \overline{BC} . Lines \overline{XB} and \overline{YC} are drawn, meeting the sides of the given triangle at V and U, respectively. If $AX = AY$ and $BV = CU$, prove that $AB = AC$.

3. Find all positive integers k such that the number $1444\dots44$, having exactly k digits equal to 4, is a perfect square.
4. Find all real numbers x that satisfy the equation

$$x = |x - |x - |x - \dots |x - |x - 1|| \dots |||$$

where there are 100 absolute values on the right hand side.

5. Let x_1, x_2, \dots, x_n be n nonnegative real numbers that sum to 1. If $n \geq 4$, find the largest possible value for the expression

$$y = x_1x_2 + x_2x_3 + \dots + x_{n-1}x_n + x_nx_1.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	Deadline January 4, 2008	
Or Email To	talent@math.wisc.edu		
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