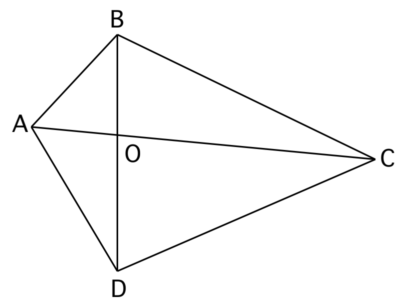
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2007-2008) **NOVEMBER 2007**

1. Let U be a set containing 29 objects and let $S_1, S_2, \ldots, S_{10}$ be 10 subsets of U, not necessarily distinct. Suppose that every 5 of the subsets taken together contain all 29 objects of U. Show that some three of these subsets taken together contain all 29 objects of U.

2. Let $ABCD$ be the convex quadrilateral, as shown, and let O be the point of intersection of its two diagonals. Suppose the area of $\triangle ABD$ is 1, the area of $\triangle BCA$ is 2 and the area of $\triangle DAC$ is 3. Find the areas of $\triangle CDB$ and $\triangle ABO$.



3. Find all positive integers a and b with $a! + 4! = b^2$.

4. For a fixed positive integer n, let $x_1, x_2, \ldots, x_n$ be n positive real numbers that sum to 1. Find the smallest possible value for the sum of the reciprocals of these n numbers.

5. Find all real numbers a such that the equation

$$|\, x - |\, x - |x - 4|\,|\,| = a$$

has exactly three real solutions x. Here, of course, $|x|$ is the absolute value of x.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Deadline December 1, 2007	
Problem	**Score**
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