

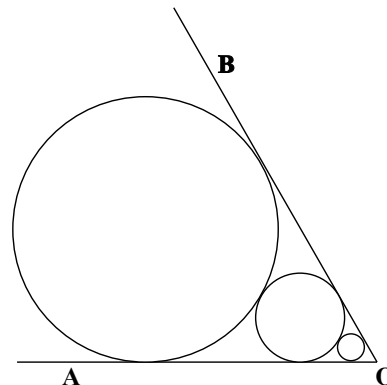
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2007-2008)

OCTOBER 2007

1. Suppose that $a \square b$ is an integer for each integer a and b . Assume that, for all integers a and b , we have $(a + 1) \square b - (a - 1) \square b = 4a$ and $b \square a = -(a \square b)$. If $1 \square 0 = 1$, find the value of $101 \square 100$.

2. Three circles are tangent to the sides of the 60° angle $\angle AOB$ and also to each other, as indicated in the diagram. If the radius of the smallest circle is 1, determine the radius of the largest circle.



3. Find all positive integers n such that the product $n(n + 16)$ is a perfect square.

4. Let x, y and z be real numbers with $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$, and $x + y + z = 2$. Prove that

$$2 \geq \frac{x}{x+1} + \frac{y}{y+1} + \frac{z}{z+1} \geq 1.$$

5. In the country of Numerica there are 10 large cities some of which are connected by highways. Each highway goes from one city to another and has no intermediate exits. Numericans never fly from one city to another if they can drive there, even if this involves traveling through several different cities. At the beginning of each year, Numerican Airlines (NA) can establish at no cost any number of direct flight routes joining two cities and it can finance road construction on any number of existing highways, thereby making those highways impassable. The profit from any flight route that anybody bothers to use is 1 million per year. The cost of financing a road construction on a single highway varies from year to year. Last year it was 5 million and NA could not make a profit despite its best efforts. This year the cost is down to 4 million. Prove that NA's profit for this year will not be greater than 9 million.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	Deadline November 1, 2007	
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