

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

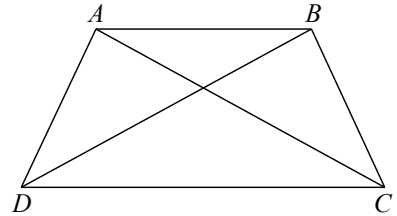
PROBLEM SET V (2006-2007)

FEBRUARY 2007

1. Let x, y and z be positive numbers such that $x + y + z \leq 1$. Show that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq 9$$

2. Let $ABCD$ be a trapezoid, with \overline{AB} parallel to \overline{CD} . Draw diagonals \overline{AC} and \overline{BD} and assume that $\angle DAC = \angle DBC$. Show that $AC = BD$.



3. Suppose that every point in the plane is colored either red or blue. Prove that there exists an equilateral triangle whose three vertices all have the same color.
4. Let us say that an infinite set S of positive integers is *anticlosed* if whenever x and y are two different members of S , their sum $x + y$ is not a member of S . Prove that the set of all positive integers is the union of some infinite collection of anticlosed subsets. Decide whether or not the set of all positive integers is the union of two anticlosed subsets.
5. Susan says that the average number of siblings among all of the children living on her street is three, but that she is a child who has more than three siblings. Find the smallest possible number of families on Susan's street. Justify your answer.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	Deadline	
Or Email To		March 7, 2007	
Please Fill In	PROBLEM SET V		
Name & Grade		1	
School & Town		2	
Home Address		3	
Town & Zip		4	
Email Address		5	