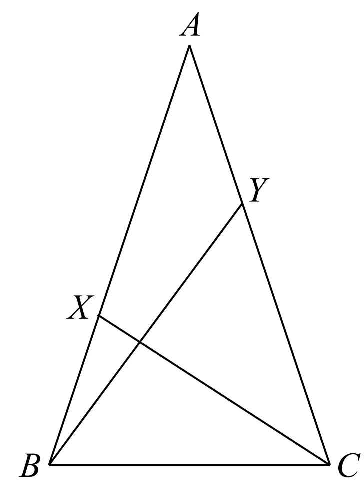
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2006-2007) **JANUARY 2007**



1. Given an integer $n > 2$, let S be the set of all integers m such that $m + n$ is a divisor of $m^2 + n^2$. Show that the set S is finite and that the number of negative numbers in S exceeds the number of positive numbers in S by at least five.

2. In the figure, $\triangle ABC$ is isosceles, with $AB = AC$ and $\angle A = 36°$. Point X on side $\overline{AB}$ and point Y on side $\overline{AC}$ are chosen so that $AX = BC = CY$. Prove that $\overline{BY}$ and $\overline{CX}$ are perpendicular.

3. Find all solutions in positive integers $a < b < c$ to the equation $(a + b + c)^2 = a^3 + b^3 + c^3$.

4. Suppose that for each integer $k \geq 1$, we have an unlimited supply of rectangular $2 \times k$ tiles. Given an integer $n \geq 1$, write $a(n)$ to denote the number of ways that a $2 \times n$ rectangle can be covered using our tiles. It is clear, for example, that $a(1) = 1$, and a little experimentation shows that $a(2) = 3$ and $a(3) = 6$. Compute $a(7)$.

5. Find all pairs of positive numbers x and y such that $x^3 - y^3 = 100$ and both $x - y$ and xy are integers.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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