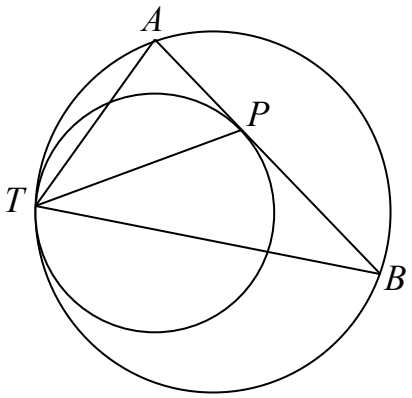
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2006-2007) **DECEMBER 2006**

1. Prove that $(x+1)(x^2+1)(x^3+1) \leq 4(x^6+1)$ for all real numbers x.

2. In the diagram, the two circles are tangent at point T, and chord $\overline{AB}$ of the large circle is tangent to the small circle at point P. Prove that $\overline{TP}$ bisects $\angle ATB$.



3. Let $a < b < c$ be positive integers such that no prime number is a divisor of all three of them. Suppose N is a positive integer that is divisible by each of a, b, and c. Prove that $N > a^{3/2}$.

4. The five diagonals of a pentagon intersect at ten points (including the five vertices of the pentagon). Find all positive integers n such that the ten numbers 1, 1, 2, 2, 2, 3, 3, 3, 4, and n can be placed on these ten intersection points in such a way that the sums of the four numbers along the diagonals are all equal.

5. What is the largest number of points that can be placed in (or on the boundary of) a 2×2 square so that the distance between each pair of points is at least 1? Justify your answer.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Deadline January 3, 2007	
Problem	**Score**
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