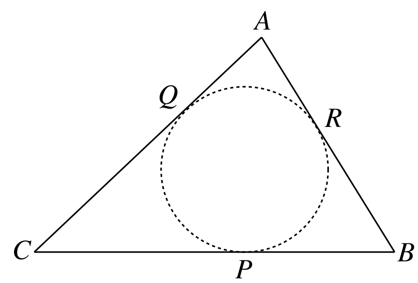
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2006-2007) NOVEMBER 2006

1. It is a fact that $2^{38} = 274877906944$, which is a number having its two rightmost digits equal. Does there exist some power of 2 whose three rightmost digits are equal? Does there exist some power of 2 whose four rightmost digits are equal? Prove that your answers are correct.

2. Points P, Q and R lie on the sides of $\triangle ABC$ as shown, with P on $\overline{BC}$, Q on $\overline{AC}$ and R on $\overline{AB}$. These three points are positioned so that $AQ = AR$, $BR = BP$ and $CP = CQ$. Prove that the inscribed circle of $\triangle ABC$ passes through points P, Q and R.



3. How many ten letter "words" are there like XXYXYYXYXX, which are composed of Xs and Ys, and which contain neither three consecutive Xs nor three consecutive Ys.

4. It is easy to check that

$$\frac{1}{8} = \frac{1}{3^2} + \frac{1}{12^2} + \frac{1}{15^2} + \frac{1}{20^2},$$

and so 1/8 is the sum of the reciprocals of four different square integers. Decide whether or not it is possible to write 1/8 as the sum of the reciprocals of *three* different square integers. Prove that your answer is correct.

5. Let n be a positive integer and let a, b and c be real numbers. Suppose that for every integer m, the quantity

$$\frac{1}{n}m^3 + am^2 + bm + c$$

is an integer. Prove that n must be one of the numbers 1, 2, 3 or 6.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To **Or Email To**	**MATHEMATICS TALENT SEARCH** Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	**Deadline** December 1, 2006
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Problem	**Score**
1	
2	
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