WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2006-2007)   **OCTOBER 2006**

1. Find all possibilities for real numbers a and b such that the polynomial $x^2 + ax + b$ has values between -2 and 2 inclusive, for all x with $0 \le x \le 4$.

2. Each side of a trapezoid is tangent to a circle of radius 1, as shown. Prove that the area of the trapezoid is at least 4. (In fact, this conclusion holds even if the given quadrilateral is not a trapezoid, but the proof seems harder in that case.)

3. Let f be a function defined for all nonzero real numbers. (Recall that this means that for each such number x, there is a uniquely determined real number $f(x)$.) Suppose that $f(1/x) + 2f(x) = x$ for all nonzero real x. Determine the function f.

4. Find all prime numbers of the form $100\cdots001$, where the total number of zeros between the first and last digits is even.

5. Eight lamps, each with an on-off switch, are arranged in a circle. A lamplighter can flip (in other words, change the state of) the switches, but he is not allowed to flip just one at a time. When he switches lamps on or off, he is required to flip the switches of three consecutive lamps simultaneously. (For example, he can flip the switches of lamps 7, 8 and 1 at the same time.) Prove that no matter what set of lamps was turned on at the start, the lamplighter can turn all the lamps on.
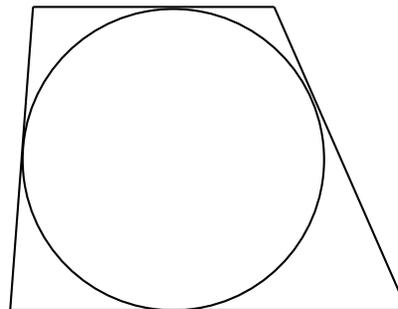
You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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